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REPORT

DIGITAL VIDEO:
assessment of comb–filter design for sub–nyquist PAL codecs

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Research Department, Engineering Division
THE BRITISH BROADCASTING CORPORATION

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N.D. Wells, B.A., D.Phil.

Summary

One very effective technique for the bit-rate reduction of digital PAL signals is sub-Nyquist sampling. This technique requires the use of comb filters which attenuate unwanted alias components produced by the sub-sampling process and also reconstruct the colour signal to standard form at the decoder. Variations are possible in the design of comb filter; in particular, the configuration can be based on line, field or picture delays.

This Report outlines the theory of comb filter operation and describes the results of subjective tests which compare the performance of the different possible sub-Nyquist systems. Certain design features are also described which are required in order that sub-Nyquist codecs may be cascaded without introducing further filtering impairment after the first such codec.

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DIGITAL VIDEO: ASSESSMENT OF COMB-FILTER DESIGN FOR SUB-NYQUIST PAL CODECS

N.D. Wells, B.A., D.Phil.

1. Introduction

For digital video transmission the bit rates of interest, at which access to the national digital telecommunications network may be made, are 34.368 Mbit/s and 139.264 Mbit/s. Techniques have been investigated which achieved a sufficient reduction in the bit rate of digitally-coded composite PAL video signals to enable a 34 Mbit/s transmission system to be defined.[1] Subsequently, a more cost-effective '68 Mbit/s' (strictly 68.736 Mbit/s) transmission system has been developed which conveys one television signal with considerable further capacity for high-quality sound channels and telephony.[2,3] Two of these 68 Mbit/s signals may be multiplexed together for transmission through a standard 139.264 Mbit/s digital link.

One of the bit-rate-reduction techniques used in these transmission systems is sub-Nyquist sampling, the principle of which has been described in previous publications.[4,5,6] In this technique, the sampling frequency is sub-Nyquist at $2f_{sc}$,* instead of at say, $3f_{sc}$, thus giving a considerable saving in bit rate. The visibility of in-band alias components produced by the sub-sampling is reduced by comb-filtering the signal both before (pre), and after (post), the sampling process. In addition to removing alias components, the comb post-filter re-constitutes chrominance information to conform to that of the standard PAL signal from the non standard signal which is actually transmitted when using the sub-Nyquist technique.[5]

Earlier work to assess the picture impairments introduced by the sub-Nyquist technique used analogue comb-filter designs. It is now simpler and more convenient to implement the filters in the digital domain which has the added advantages of freedom from drift and no in-band, group-delay distortion.

The comb filter frequency response is traditionally obtained by averaging the high-frequency signal information across a one-line-period delay. However, this delay may alternatively be of 'field' (313 lines) or 'picture' duration.[6] The length of the delay affects the type of picture impairment introduced by the comb filters. For example, line-delay-based comb filters introduce a loss of high-frequency luminance diagonal resolution and also a slight loss of vertical chrominance resolution. For stationary pictures, picture-delay-based comb filters do not introduce any loss of resolution. However, when averaging across a picture delay they inevitably cause a loss of resolution with picture movement.

Experimental, digitally-implemented comb filters were constructed in which the delay element could be a line, field, or picture period in duration. A series of subjective tests was then carried out to compare the relative picture impairments introduced by these different types of filter on critical stationary and critical moving pictures. This Report describes the design considerations, the experimental digital comb filter implementation, and gives the results of subjective tests. In addition, the quantisation noise introduced by digital implementation is discussed and an 'optimum' design of filter, which minimises the impairment caused by cascaded sub-Nyquist codings and decodings is described.

The Report starts with a brief explanation of the effect of line-, field-, and picture-delay comb filters on the PAL spectrum.

2. The PAL spectrum

2.1. The luminance spectrum

As a result of the repeated scanning process the frequencies present in a video signal for a stationary scene occur only at discrete intervals.[7,8] The signal frequency corresponding to a spatial frequency of m cycles/picture width (c./p.w.) and n cycles/picture height (c./p.h.) may be calculated by considering the scanning spot sweeping, with a velocity v_x picture widths/sec and v_y picture heights/sec, across a spatial frequency grating defined by (m, n). Then, the signal frequency f(Hz) is given by

$$f = \mathrm{m}v_x + \mathrm{n}v_y$$

For an interlaced scanning standard with N lines/picture and a line scan frequency of f_L(Hz), v_x (p.w./s.) and v_y (p.h./s.) are given by

$$v_x = f_L \qquad \text{and} \qquad v_y = \frac{-2f_L}{\mathrm{N}}$$

---

* f_{sc} = colour subcarrier frequency = 4.43361875 MHz for PAL signals.

Then
$$f = \left(m - \frac{2n}{N}\right) f_L \qquad (1)$$

(The negative sign occurs because the scanning proceeds in the negative y direction.)

The luminance spectrum for still pictures is then of the form shown in Fig. 1 (which uses, for illustration, a value of N equal to 15). It can be seen that the spectrum has components which fall at multiples of picture frequency, f_L/N, only.

2.2. Chrominance spectrum

As both the *U* and *V* components, before modulation, are derived from the same scanned source as the luminance, the form of their spectra will also be of the form shown in Fig. 1. In order to obtain the phase alternation in the final *V* subcarrier, the *V* signal is first multiplied by a square wave of half line-frequency before double-sideband, suppressed-carrier modulation. Figs. 2(a) and 2(b) show the forms of the *U* and *V* spectra after quadrature modulation of the colour subcarrier frequency.

2.3. Motion

When there is motion in a scene, the frequencies present in the video signal no longer lie only at multiples of the picture frequency. A spatial frequency defined by m c./p.w. and n c./p.h. moving with velocity of u_x picture widths/sec and u_y picture

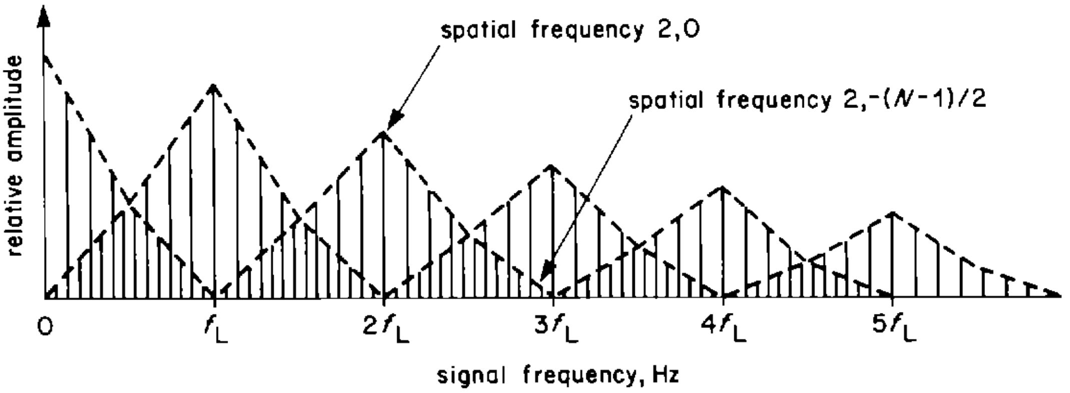

Fig. 1 – Luminance spectrum of a typical interlaced scanned television signal.

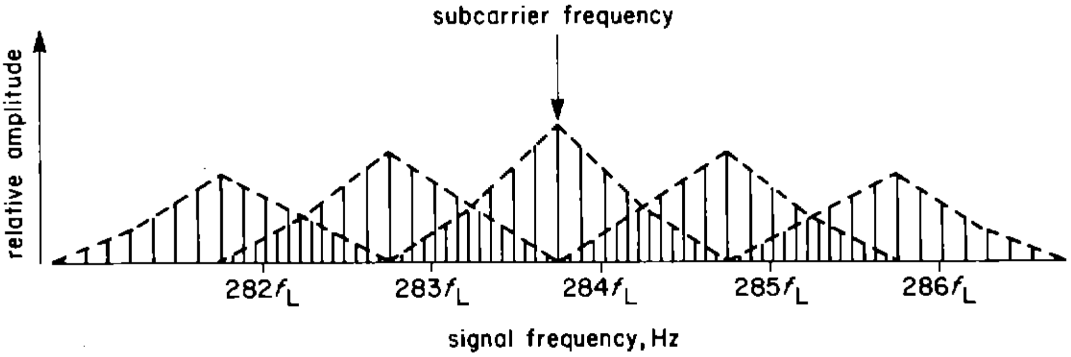

Fig. 2(a) – Spectrum of U chrominance signal.

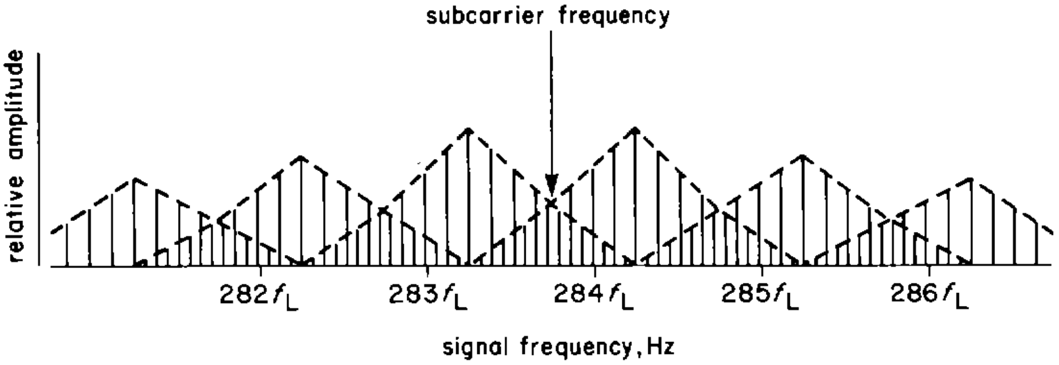

Fig. 2(b) – Spectrum of V chrominance signal.

heights/sec will give a signal frequency f where

$$f = m(v_x - u_x) + n(v_y - u_y)$$

Since for the same component in a stationary scene

$$f_{\text{stationary}} = mv_x + nv_y$$

$$f - f_{\text{stationary}} = -(mu_x + nu_y) \qquad (2)$$

Thus the shift in frequency of a given signal component depends on the spatial frequency (m, n) and the speed and direction of motion.

2.4. Spectrum of the sub-Nyquist-sampled signal

When a PAL signal is sampled at $2f_{sc}$, a signal frequency, f, can give rise to a higher order or 'alias' component at a frequency, $2f_{sc} - f$, which falls within the baseband signal bandwidth. Since

$$f_{sc} = (283 + \tfrac{3}{4})f_L + f_p$$

where f_L = line frequency and f_p = picture frequency,

$$2f_{sc} = (567 + \tfrac{1}{2})f_L + 2f_p$$
$$= (354689 + \tfrac{1}{2})f_p \qquad (3)$$

Thus, $2f_{sc}$ is exactly equal to an odd multiple of $\frac{1}{2}f_p$ and, as $f_L \gg f_p$, is approximately equal to an odd multiple of $\frac{1}{2}f_L$.

For the luminance component, high frequency signals which fall predominantly at multiples of line frequency will give rise to alias components falling approximately half-way in between the line frequency harmonics. On a fine scale, all signal components (for still pictures) at multiples of picture frequency, f_p, will give alias components falling at odd multiples of $\frac{1}{2}f_p$.

For the chrominance signal, the modulated U and V components each have a sideband structure which is symmetrical* about f_{sc} and, on sampling at $2f_{sc}$, an upper sideband will give rise to an alias component at the frequency of the corresponding lower sideband and vice-versa.

The phase and amplitude of the resulting chrominance signal are a function of the sampling phase. In a sub-Nyquist PAL transmission system the sampling phase is locked in relation to the U subcarrier phase and is chosen such that the original signal and alias components reinforce to compensate for loss at the colour-subcarrier frequency introduced by the comb filtering. This is discussed fully in Refs. 4 and 5.

For a System I PAL signal, the signal frequencies extend up to 5.5 MHz which, on sampling at $2f_{sc}$, give alias components with frequencies down to 3.3 MHz. Comb filtering of the signal is only necessary therefore for the frequency range 3.3 MHz to 5.5 MHz.

3. Comb filters

3.1. General

Comb filters reduce the amplitude of the alias components produced by sub-sampling and are generally used in pairs with a pre-filter prior to sampling and a post-filter following. The comb post-filter attenuates alias components while minimising the effect on the original signal components. The comb pre-filter removes those signal components which on sampling would give rise to alias components which are not removed by the post-filter.

Comb filters are constructed most simply by averaging the high signal frequencies (above 3.3 MHz) across a single delay, T, as shown in Fig. 3. The designs of the pre- and post-filter differ slightly in order to obtain a uniform group delay (equal to T) over the entire baseband spectrum for the filter pair. The length of the delay element can be equal to one line, one 'field' (313 lines) or one picture period. The effects of these types of filter on the luminance and chrominance components are discussed in the following two Sections.

Comb filters using several delay elements are also possible.[5] The shape of the teeth of the filter response can thereby be changed to improve the separation between wanted and alias components. These more complicated filter designs are not necessarily 'cascadable' (as described in Section 6) and are not considered in this Report.

3.2. Comb filtering of the luminance component

3.2.1. Line-delay-based comb filters

Fig. 4 shows the high-frequency amplitude response of a single comb filter between adjacent harmonics of line frequency. For later comparison with field- and picture-delay filters, this figure indicates the positions of spectral lines from stationary diagonal detail. For illustration, the separation of these lines has been expanded relative to line

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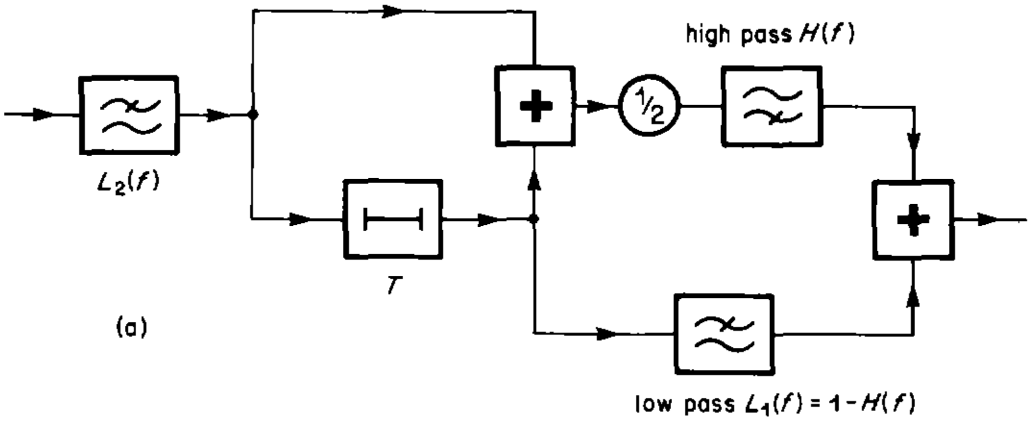
*Strictly speaking, the sidebands are only symmetrical for about ±1.1 MHz.



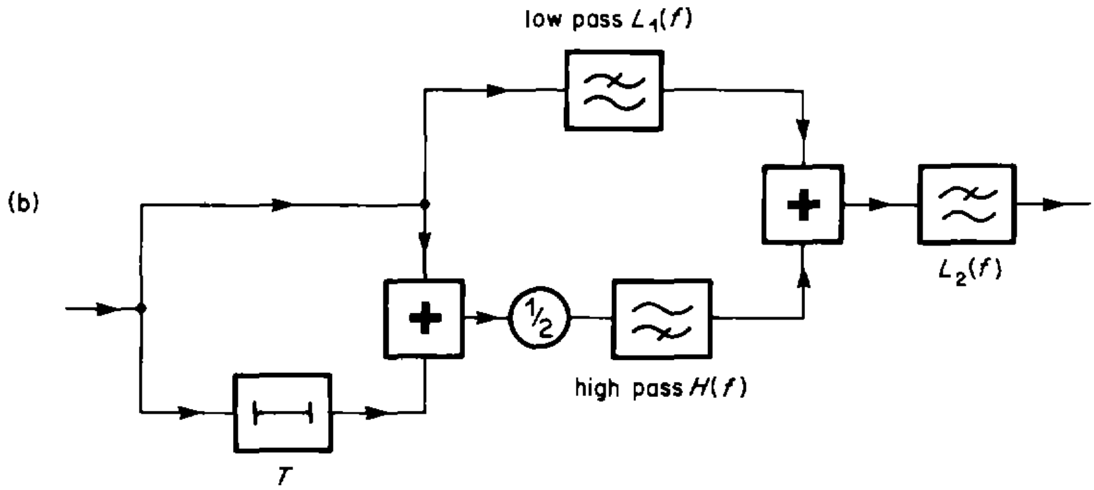


Fig. 3(a) – Comb pre-filter; (low frequencies delayed by T, high frequencies delayed by T/2).
Fig. 3(b) – Comb post-filter; (low frequencies undelayed, high frequencies delayed by T/2).

frequency. The spacing shown would result from interlaced scanning with 21 lines/picture.

Fig. 5 shows the vertical frequency amplitude response of a comb-filter pair for high-frequency components above 3.3 MHz. (The signal is not filtered below 3.3 MHz.) The response has a null at 156 c./p.h. The dotted curve in Fig. 5 shows the maximum amplitude of the residual luminance alias component after sampling and filtering for a given input vertical frequency within the combed band. The alias component will have a frequency different from that of the input frequency and can give rise to a visible moving pattern caused by beating between the input and alias components. This alias patterning is most visible for frequencies near colour-subcarrier frequency.

3.2.2. 'Field'-delay-based comb filters

When the delay element, T, in the filters is equal to a field (313 line) period the separation of the 'teeth' of the filter response is equal to the field frequency. This is shown in Fig. 6 which again uses for illustration the example of a scanning system with 21 lines/picture. It can be seen that for a stationary picture the nulls in the filter response fit mainly between the main spectral components. If N is the number of scanning lines/picture then the separation of the spectral components for still pictures is $2f_L/N$ whereas the separation of the 'teeth' of the comb response is $2f_L/(N + 1)$. This gives rise, in the combed band, to the vertical frequency response shown in Fig. 7 for a comb-filter pair for a 625-line system. This figure shows that field-delay-based filters with a null in the amplitude response around

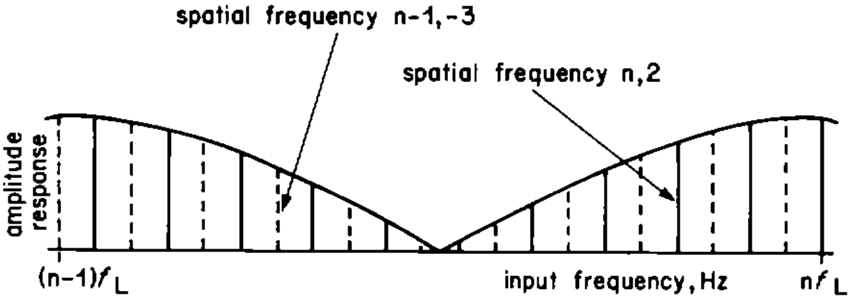


Fig. 4 – High-frequency amplitude response of a single line-delay comb filter.

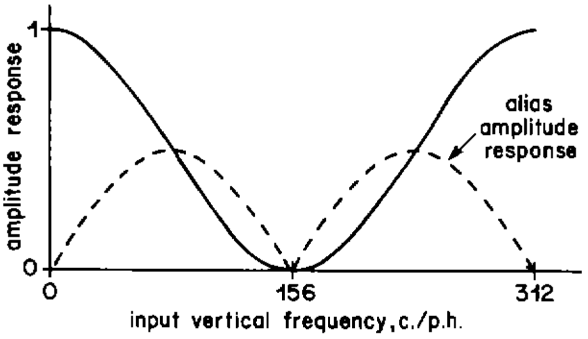


Fig. 5 – High-frequency amplitude response as a function of vertical frequency for a line-delay comb-filter pair. The dotted curve shows the amplitude response for residual alias components.

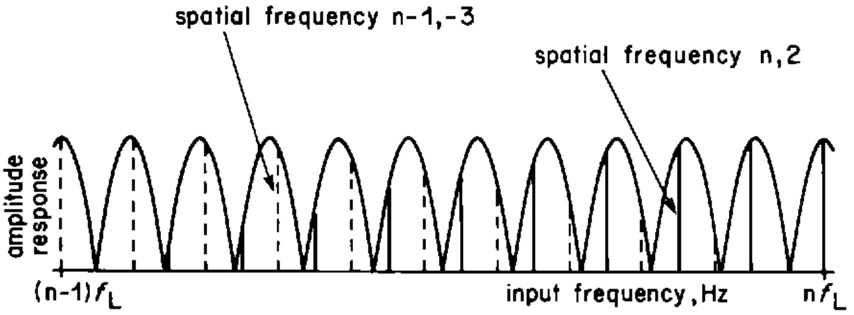


Fig. 6 – High frequency amplitude response of a single, 'field'-delay comb filter.

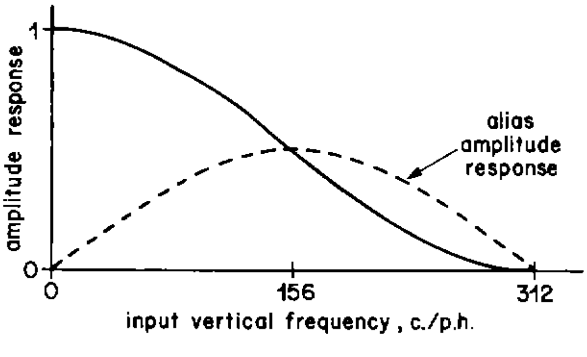


Fig. 7 – High-frequency amplitude response as a function of vertical frequency for a 'field'-delay comb-filter pair with stationary pictures.

312 c./p.h. give a better vertical frequency resolution in the combed band than line-delay-based filters.

In order to remove luminance alias components after $2f_{sc}$ sampling the delay element must equal 313 lines rather than say 312 lines. Frequencies which lie at or near line-frequency harmonics give alias components at and around half-line-frequency positions (or more precisely half-line frequency plus twice picture frequency) and these will be removed by the post filter. A 312-line filter would have a maximum at the half-line-frequency position and would not attenuate the alias components. The dotted curve in Fig. 7 gives the residual alias amplitudes in the combed band as a function of the input vertical spatial frequency for stationary pictures.

It may also be seen from Fig. 6 how field-delay comb filters reduce resolution with picture movement. A spectral component at a line frequency harmonic which is shifted by 25 Hz due to picture motion will be completely suppressed by the filters. For example, Equation (2) may be used to show that 4 MHz frequency bars will be suppressed at a motion speed of approximately $\frac{1}{10}$ picture widths/sec. Comparing this with line-delay filters (Fig. 4), a 25 Hz shift would cause little change in attenuation of a particular component and hence little change in resolution.

Also, for field-delay filters, picture movement will change the amplitude distribution of the residual alias components after the post filter. These will be at a maximum for input frequencies which are such that they are attenuated by 3 dB by the pre-filter (giving rise to alias components which are attenuated by 3 dB by the post filter).

3.2.3. Picture-delay-based comb filters

In the combed region of the response of picture-delay comb filters the 'comb' maxima are at multiples of picture frequency. Since all luminance spectral components for stationary pictures also lie only at multiples of picture frequency, a picture-delay comb filter has no effect on the signal. This is illustrated in Fig. 8. On sampling, all alias components will fall at odd multiples of half picture frequency and will be removed by the post filter.

For stationary pictures, therefore, there is no need for a pre-filter. However, a pre-filter is required for moving scenes to ensure an overall uniform group delay of one picture period and also to remove input frequencies, in particular those shifted by $12\frac{1}{2}$ Hz, which would give alias components which would not then be attenuated by the post filter.

3.3. Chrominance response in sub-Nyquist systems

As the chrominance information is modulated onto a subcarrier falling at the centre of the combed region, all (or virtually all) the chrominance is averaged across the delay of the comb filter. Since the modulated spectral components do not lie at multiples of line or 'field' or picture frequency, as shown in Fig. 2, these components will be attenuated by the filters. However, the phase of sampling at the analogue-to-digital converter can be chosen such that the alias components add in phase with the original components and compensate for the filter loss in areas of uniform colour.[4,5]

Briefly, the sampling phase is chosen as follows. The pre-filter causes the U subcarrier to become co-phased with the V subcarrier (in uniform coloured areas). The chrominance components are also attenuated giving a subcarrier amplitude of $(U + V)/\sqrt{2}$ on one line and $(U - V)/\sqrt{2}$ on the next. The sampling instants at the a.d.c. are then arranged to lie at the peaks and troughs of this 'unified' chrominance component. When this sampled and digitised signal is reconverted to analogue form and low pass filtered (to 5.5 MHz) the subcarrier has an amplitude equal to $(U \pm V) \times \sqrt{2}$ because of the in-phase addition of the baseband and alias components. This enhanced subcarrier is then attenuated and reconstituted into standard PAL form, with U and V in quadrature, by the post-filter.

As with the luminance component, Fig. 5 and Fig. 7 also give the effect on vertical chrominance resolution for line- and field-delay-based filtering (a detailed analysis of this is given in Appendix A). The filtering and sampling process also introduces U/V cross-talk on horizontal chrominance edges. The magnitude of this cross-talk as a function of vertical frequency is also derived in Appendix A and shown

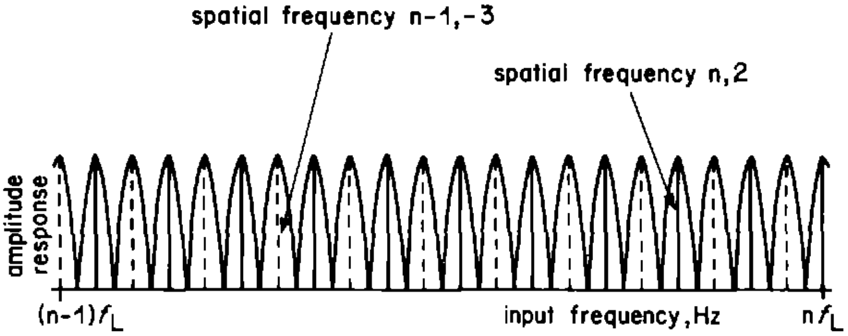


Fig. 8 – High-frequency amplitude response of a single picture-delay comb filter.

in Fig. 9. Sub-Nyquist systems using picture-delay-based filters do not introduce any chrominance resolution loss or U/V cross-talk for stationary pictures. However, impairments appear on moving picture detail.

3.4. Cross-colour

The phenomenon known as 'cross-colour' occurs because high-frequency luminance information is interpreted as chrominance information by the PAL decoder. In a sub-Nyquist system, the comb filters attenuate, to some extent, luminance components that give rise to the coarsest cross-colour. On the other hand, the $2f_{sc}$ sampling process generates alias components that can enhance the amplitude of the cross effects. On balance, both line- and field-delay-based sub-Nyquist systems reduce the level of cross-colour impairment. Picture-delay-based systems do not affect cross-colour with stationary pictures.

4. Digital comb filters

4.1. General

In order to implement the comb filter digitally at the coder the signal is first sampled at a super-Nyquist rate. The required output sample values at the sub-Nyquist rate are then calculated by a digital interpolating filter. An excellent exposition of the theory of interpolating filters is given in the introductory sections of Reference 9. At the decoder, samples at the required output rate are produced, again by an interpolating filter, from the $2f_{sc}$ input values.

The implementation of the interpolating filters is comparatively simple if there is a precise 2:1 ratio between the input and output sample rates. Accordingly, for a digital comb pre-filter, the PAL signal is first sampled at $4f_{sc}$ and an appropriately-weighted sum of several input samples is formed to give comb-filtered output values also at a $4f_{sc}$ rate. The signal is then sub-sampled to $2f_{sc}$ by discarding alternate samples. The digital comb post-filter forms the missing $4f_{sc}$ sample values by interpolating between the $2f_{sc}$ input samples. The form of the interpolation can be chosen to give the required comb-filter response. Finally the $4f_{sc}$ signal may be re-converted back to analogue and low-pass filtered in the normal way to remove the higher-order spectra centred on $4f_{sc}$, $8f_{sc}$, etc. The whole process is outlined in Fig. 10. The boxcar equaliser shown after the d.a.c. is discussed in Section 4.2.

4.2. Experimental implementation

An experimental digital comb filter pair was constructed in the form shown in Fig. 11. This form of design is functionally equivalent to the form shown in Fig. 3 and uses one bandpass filter to define the combed frequency band rather than a combination of high-pass and low-pass filters.[4,5,10] The

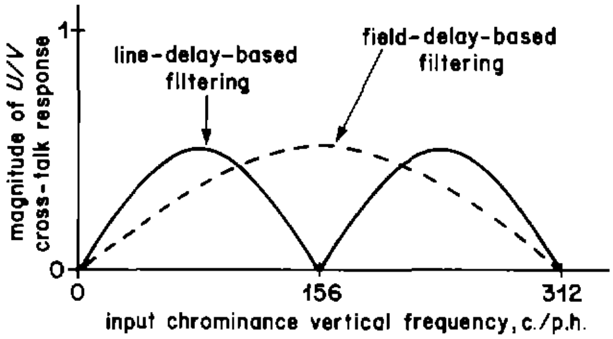


Fig. 9 – Magnitude of U-V cross-talk response as a function of vertical chrominance frequency.

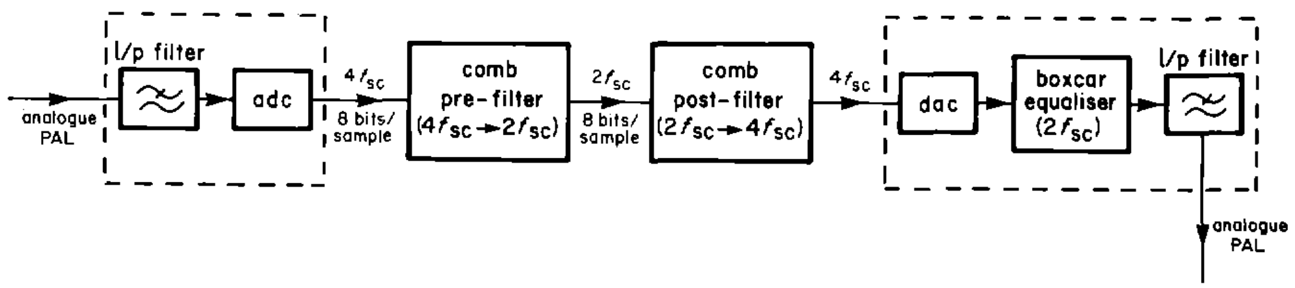

l/p filter
analogue PAL
adc
$4f_{sc}$
8 bits/ sample
comb pre-filter ($4f_{sc} \rightarrow 2f_{sc}$)
$2f_{sc}$
8 bits/ sample
comb post-filter ($2f_{sc} \rightarrow 4f_{sc}$)
$4f_{sc}$
dac
boxcar equaliser ($2f_{sc}$)
l/p filter
analogue PAL

Fig. 10 – Outline of sub-Nyquist system with digital comb filters.

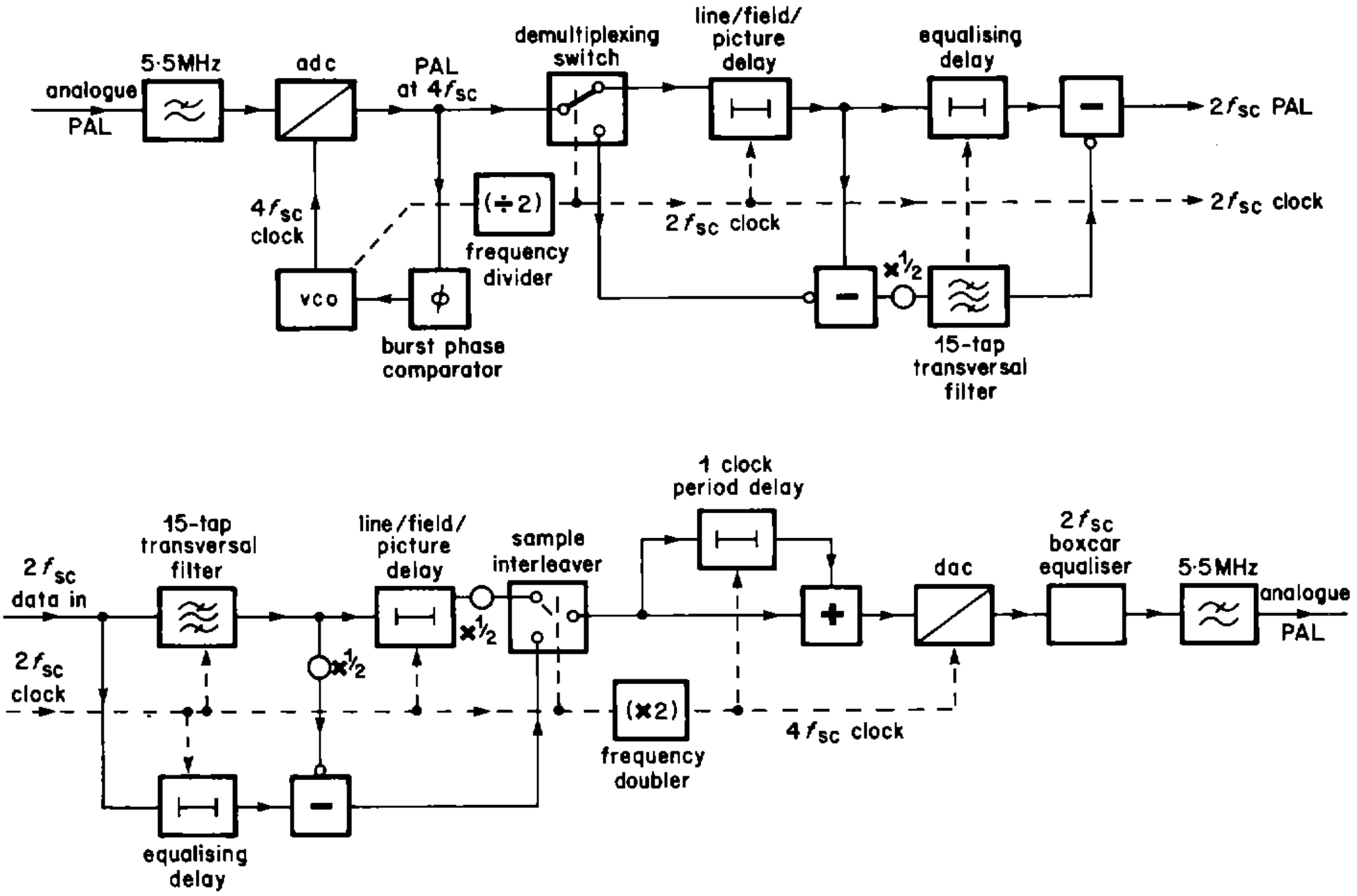


Fig. 11 – Block diagram of experimental digital comb filters: (a) comb pre-filter; (b) comb post-filter.

filters are organised such that as much of the digital processing as possible is done at a clock rate of $2f_{sc}$ rather than at the awkwardly high clock rate of $4f_{sc}$.

Before the comb pre-filter, the analogue PAL signal is converted to a $4f_{sc}$ sampled, p.c.m. signal with sampling instants adjusted to be at 45°, 135°, 225° and 315° with respect to the maxima of the modulated U chrominance component. This $4f_{sc}$ signal is demultiplexed into two $2f_{sc}$ data streams, one containing 45° and 225° samples and the other containing 135° and 315° samples. In the case of line-delay based filters the 45° and 225° samples are delayed by $567\frac{1}{2}$ $2f_{sc}$ clock periods (0.4 ns less than one line period) bringing them into phase with the undelayed 135° and 315° samples from the following line period. The two data streams are then combined to give the required combed frequency response. In the case of field-delay based filters it is the 135° and 315° samples that are delayed (by $177\,628\frac{1}{2}$ $2f_{sc}$ clock periods) relative to the 45° and 225° samples. With picture-delay based filters the 45° and 225° samples are again delayed (by $354\,688\frac{1}{2}$ clock periods) relative to the other samples. These choices of sampling phase are discussed in the next Section.

In the post filter two $2f_{sc}$ data streams, derived from alternate lines (fields or pictures), are processed as shown and multiplexed to give one $4f_{sc}$ data stream. The two data streams have different d.c. levels and, in order to remove the large resulting $2f_{sc}$ component in the $4f_{sc}$ data, the signal is averaged across a $4f_{sc}$ sample-period delay which acts as a filter with a null at $2f_{sc}$. The d.a.c. at the output of the post filter is followed by a boxcar equaliser which compensates for the high frequency loss due to the finite period for which each sample value is held. In this case, the combination of the sample-averaging filter with a null at $2f_{sc}$ together with the $4f_{sc}$ boxcar loss is equivalent to $2f_{sc}$ boxcar loss. Therefore, $2f_{sc}$ boxcar equalisation is used even though the d.a.c. is working at a $4f_{sc}$ sample rate.

The amplitude response of the comb filter pair is shown in Fig. 12. The upper band edge of the combed region corresponds to the response at multiples of line (or field or picture frequency) and is defined by the two analogue low-pass filters associated with the a.d.c. and d.a.c. The lower edge of the combed band corresponds to the response of the comb filters at odd multiples of half-line (or half-field or half-picture) frequency and is defined by the frequency response of the digital band-pass filters. The frequency response of Figure 12 is not ideal because of the finite attenuation of odd multiples of half-line frequency at around 6 MHz.

If $B(f)$ is the frequency response of the digital band-pass filter, then the response of the comb filter pair at odd multiples of half-line (or half-field etc.) frequency within the combed band is given by $(1 - B(f))^2$. Because, in the designs of Fig. 11, the digital bandpass filters are operated at a clock rate of $2f_{sc}$, their response, $B(f)$, is necessarily symmetrical about f_{sc}, giving an overall comb-filter frequency response as shown in Fig. 12. The size of the 'hump' around 6 MHz could be reduced by widening the pass band of the band-pass filter but this would give an excessive loss of diagonal luminance resolution.

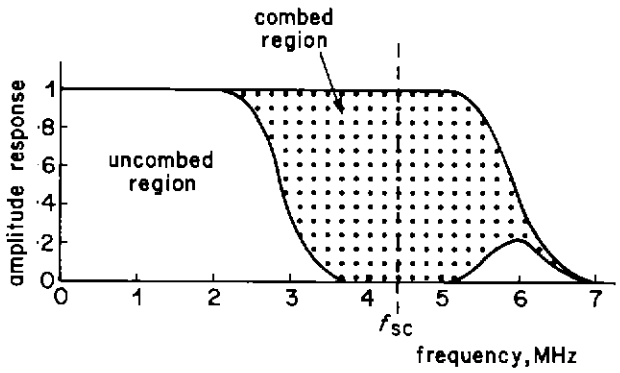


Fig. 12 – Response of digital comb-filter pair.

The type of response shown gives rise to a slight increase in the amplitudes of alias components at half-line (or field, etc.) frequencies above 5 MHz but these are not visible for a single sub-Nyquist coding and decoding. However, with a second (cascaded) sub-Nyquist coding, these alias components are re-inforced by further alias components at the same frequency caused by the second sampling process. In addition, the unattenuated alias components at half-line frequencies from the first sampling, themselves give rise, on sampling a second time, to alias components at lower frequencies at multiples of line frequency. It can be shown that these second-order aliases add in phase with the original signal components giving peaks in the frequency response. These effects are re-inforced by each subsequent sub-Nyquist process. A design modification which overcomes these problems is described in a later section.

4.3. Sampling phase

Before the pre-filter, the PAL signal is converted into a PCM signal at a sampling rate of $4f_{sc}$ with sampling instants adjusted to be at 45°, 135°, 225° and 315°, with respect to the maxima of the U chrominance component. In the experimental equipment this was achieved by locking the sampling phase to the PAL colour burst so that the $4f_{sc}$ sampling instants lay at the maxima and minima and zero-crossings of the burst.

These particular sampling phases may be understood by comparison with the analogue pre-filter case using the example of a line-delay-based filter. On passing the PAL signal through a filter of the form shown in Fig. 3(a), the phase of the U subcarrier component at a fixed point on a given line is *retarded* by 45°. Therefore, if, after the filter, this point lies at a U-subcarrier maximum then, prior to the filter, the phase of subcarrier at this point on the line would have been 45° after the maximum. This then defines the $4f_{sc}$ sampling positions on a given line since, as mentioned in Section 3.3, half of the sample positions must lie at the maxima and minima of the U-subcarrier after the filter.

For a field-delay-based comb filter the U-subcarrier phase at a fixed point on a given line is *advanced* by 45° by the pre-filter. Compared with the line-delay example, the $4f_{sc}$ sample positions are thus unchanged, but the $2f_{sc}$ sample positions selected at the output correspond to the positions which were discarded in the line delay case.

5. Noise performance

5.1. General

When a signal is sampled at f_s and quantised with a quantisation step Δ, the noise power σ^2 introduced in the band 0 to $\frac{1}{2}f_s$ Hz is given by $\sigma^2 = \Delta^2/12$.[11,12] The quantising noise spectral density is then $2\sigma^2/f_s$, a value which is inversely proportional to the sampling frequency. Thus, when we sub-sample a $4f_{sc}$ data stream by selecting alternate samples the quantising noise spectral density is doubled.[*,12]

On the other hand, the total quantising noise is attenuated by the pre- and post-filters. Over the combed frequency band each comb filter, which has sinusoidal teeth in its response, reduces the noise power on average by a factor of two. The net effect of $4f_{sc}$ sampling, $2f_{sc}$ sampling, quantisation and comb filtering is described below.

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[*] Strictly speaking the noise spectral density is doubled for a normalised value of the signal after digital-to-analogue conversion.

5.2. Noise performance with digital comb filters

Quantising noise enters the system at three points:

1. Quantisation of the $4f_{sc}$ sampled digital.
2. Truncation of the $2f_{sc}$ sample values after comb pre-filtering.
3. Truncation of the post-filtered, $4f_{sc}$ signal before the d.a.c.

Fig. 13 illustrates the build-up of quantising noise power through a sub-Nyquist transmission system using digital comb filters. This figure is drawn on a scale corresponding to a practical case with 8-bit quantising accuracy for the a.d.c. and transmission channel and 10-bit accuracy at the d.a.c. The total noise power for this case is then given approximately by the expression

$$\frac{\Delta^2}{12}\frac{[2f_b + 0.75(f_m - f_b)]}{f_{sc}}$$

where Δ is the quantising interval corresponding to 8-bit quantising accuracy. f_m is the maximum signal frequency and f_b is the frequency of the lower band edge of the combed region which is equal to $2f_{sc} - f_m$.

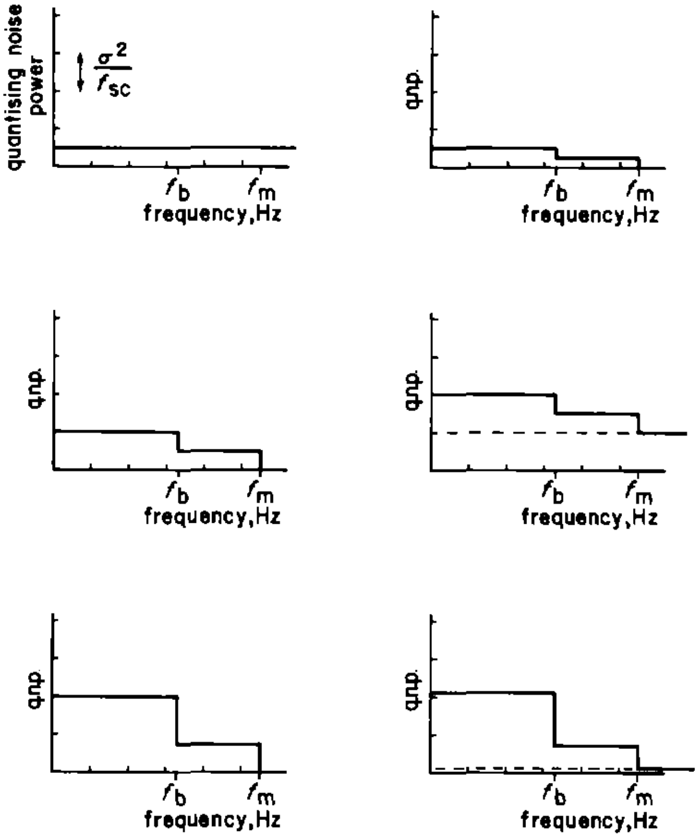

Fig. 13 – Illustration of build-up of quantising noise power in a $2f_{sc}$ digital transmission system [$\sigma^2 = \Delta^2/12$ where Δ = 8-bit quantum step]:

(a) at output ($4f_{sc}$) of a.d.c.
(b) at output ($4f_{sc}$) of comb pre-filter
(c) after sub-sampling to $2f_{sc}$
(d) truncation to 8-bits (at $2f_{sc}$) for transmission
(e) after comb post-filter
(f) after truncation to 10-bits (at $4f_{sc}$) before d.a.c.

For an 8 bit coding of a PAL signal, Δ is approximately equal to 0.5 mV. Then a peak white signal of 0.7 V corresponds to 140Δ. The unweighted signal/noise power ratio at the output of the system is then given by

$$S/N = 10\log_{10}\left[\frac{(140)^2 \cdot 12f_{sc}}{2f_b + 0.75(f_m - f_b)}\right] \text{dB}$$

Taking $f_m = 5.5$ MHz and $f_b = 3.3$ MHz, this approximate formula gives a value of 51.0 dB for the *unweighted* video signal-to-noise ration for a typical digital sub-Nyquist transmission system. Weighting the noise according to CCIR Recommendation 567-1 gives a value of 57.8 dB.

6. Multiple sub-Nyquist coding

The slight impairments produced by sub-Nyquist coding do not necessarily build up with successive (cascaded) coding and decoding operations. The effect of two codecs in cascade is no greater than the effect of a single codec provided that the overall filter response between successive sub-samplings has a uniform group delay and has an amplitude versus frequency response which satisfies the condition:

$$A(f) + A(2f_{sc} - f) = 1 \qquad (4)$$

Stott and Phillips[13] give a derivation of this result and also show that the fine comb-structure of the response of a comb post-filter followed by a comb pre-filter satisfies this condition. However, they assume a sharp transition boundary between the combed and uncombed frequency bands. For equation (4) to be satisfied at all frequencies including frequencies falling at the boundary between the combed and uncombed regions, the overall shapes of the band edges also have to satisfy this condition. Referring back to Fig. 3, if $L_1(f)$ is the amplitude response of the low-pass filter defining the lower band edge and $L_2(f)$ is the amplitude response of the low-pass filter defining the upper band edge, then

Equation (4) is satisfied only if

$$L_1{}^2(f) + L_2{}^2(2f_{sc} - f) = 1 \qquad (5)$$

This is illustrated in Fig. 14 and proved in Appendix B. Most probably, $L_2(f)$ will be determined by the analogue filters in the a.d.c. and d.a.c. The required frequency response for $L_1(f)$ defined by

$$L_1(f) = [1 - L_2{}^2(2f_{sc} - f)]^{\frac{1}{2}}$$

must then be matched as well as possible by tailoring the response of the transversal band-pass (or high-pass) filter in Fig. 11.

Using the experimental comb filters it was found that a sharp cut-off response for $L_1(f)$ tended to give ringing on diagonal luminance detail for line-delay filters.* Therefore an optimum choice of $L_1(f)$ and $L_2(f)$ is a compromise, balancing diagonal resolution loss against diagonal ringing.

The response shown in Fig. 12 for the experimental filters does not satisfy Equation (4) at all frequencies, particularly because of the hump in the combed response above 5 MHz. In order to avoid this form of response the filters would need to work at a $4f_{sc}$ clock rate throughout which is rather fast for some of the arithmetical operations required. However, since samples at the output of the pre-filter and at the input to the post filter occur only at $2f_{sc}$, it is possible to reduce the clock rate in critical parts of the circuit. Fig. 15 and Fig. 16 give designs for the pre- and post-filters which would give cascadable sub-Nyquist filtering. In the pre-filter, the section performing the band-pass filter function is working with input samples at a $4f_{sc}$ data rate but is required to calculate only alternate output samples as a $2f_{sc}$ data stream. The transversal band-pass filter may thus be configured as shown in Fig. 15. The coefficient set of filter A, interleaved with the coefficient set of filter B, equals the coefficient set of the equivalent filter C. At the post-filter the $2f_{sc}$ data input may be considered as a $4f_{sc}$ data stream with alternate samples set equal to zero. This again allows the band-pass filter to be implemented in two sections each operating at a $2f_{sc}$ clock rate as shown in Fig. 16.

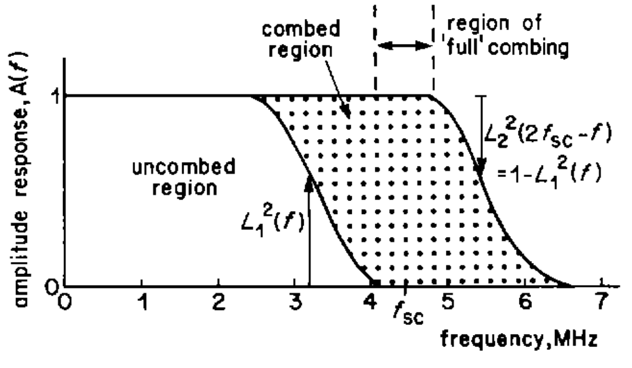

Fig. 14 – Frequency response of comb-filter pair satisfying 'anti-symmetry' condition.

7. Subjective testing of line-, field- and picture-delay comb filtering

7.1. General

This Section describes the results of subjective tests in which the picture quality was assessed for sub-Nyquist systems using comb pre- and post-filters based on line, field, and picture delays. Both static and moving picture material was used. The tests were done with PAL signals which were coded both with, and without, a 'notch' in the response of the luminance channel at f_{sc}†. These signals are described as 'notched PAL' and 'un-notched PAL' respectively.

The test material was examined for the following impairments:

(i) Reduction in high-frequency (diagonal) luminance resolution and luminance patterning caused by residual alias components.

(ii) Reduction in vertical chrominance resolution and U/V cross-talk.

(iii) Loss of resolution and patterning on moving picture material.

7.2. Test arrangements

The static pictures were obtained from a high-quality, 35 mm colour slide scanner.

The moving pictures were obtained from a high-quality digital tape recording of a studio camera output. In making the recording, the analogue R, G and B camera outputs were first converted to a 12:4:4 MHz sampling format for the digital YUV signal. The U and V signal bandwidths were defined by a Gaussian filter which was 6 dB down at 2 MHz. On replay the digital YUV was reconverted to RGB prior to PAL coding. Both 'notched' and 'un-notched' PAL coders were used.

The comb filters were instrumented digitally as described in Section 4.

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*The ringing appeared on moving detail with picture-delay based filters.

†The notch filter had an attenuation of 12 dB at subcarrier frequency and a 3 dB width of 1.5 MHz.

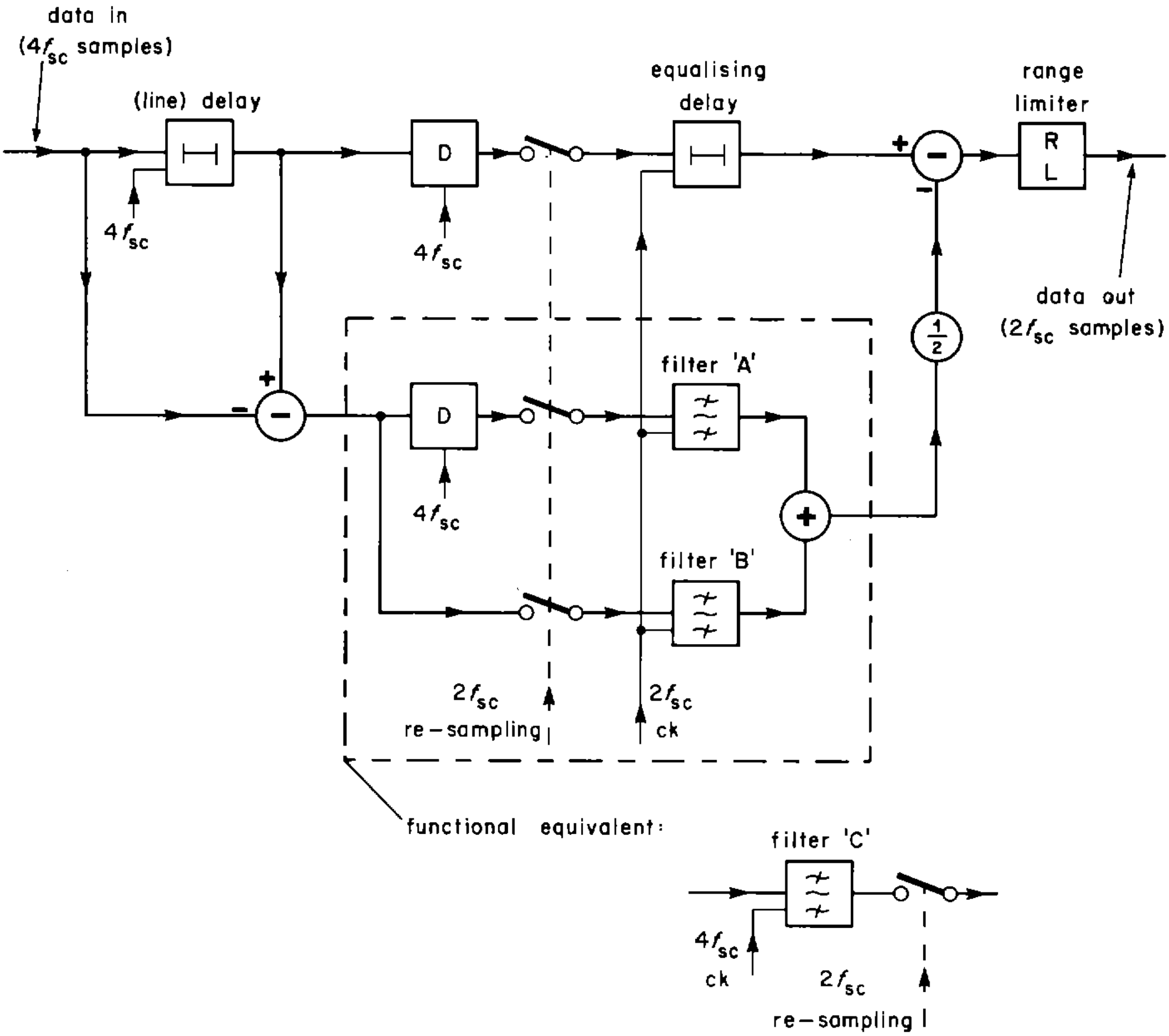

Fig. 15 – Functional block diagram of a comb pre-filter suitable for cascaded codings and decodings.

The pictures were displayed on a 22″ colour monitor. For some of the tests the pictures were viewed in monochrome on the same monitor. In the monochrome mode, the frequency response of the monitor luminance channel did not have a notch at colour subcarrier frequency.

7.3. Test pictures

For the stationary picture tests, the following standard test slides were shown:

(i) 'Test card F'
(ii) 'Boats'
(iii) 'Blackboard and toys'
(iv) 'Young couple'

All these slides were chosen as they were particularly revealing of the comb-filter resolution losses and also of the presence of residual alias components from the sub-Nyquist sampling. In the first test, however, the slide of 'Clown' was used instead of 'Young couple'. The slide was changed after the first test because it was thought that the 'Young couple' picture provided a more critical test of the sub-Nyquist system.

Two movement sequences were used. These were:

(i) Model train; this sequence consisted of a close-up shot of a model locomotive and several carriages moving fairly rapidly across the

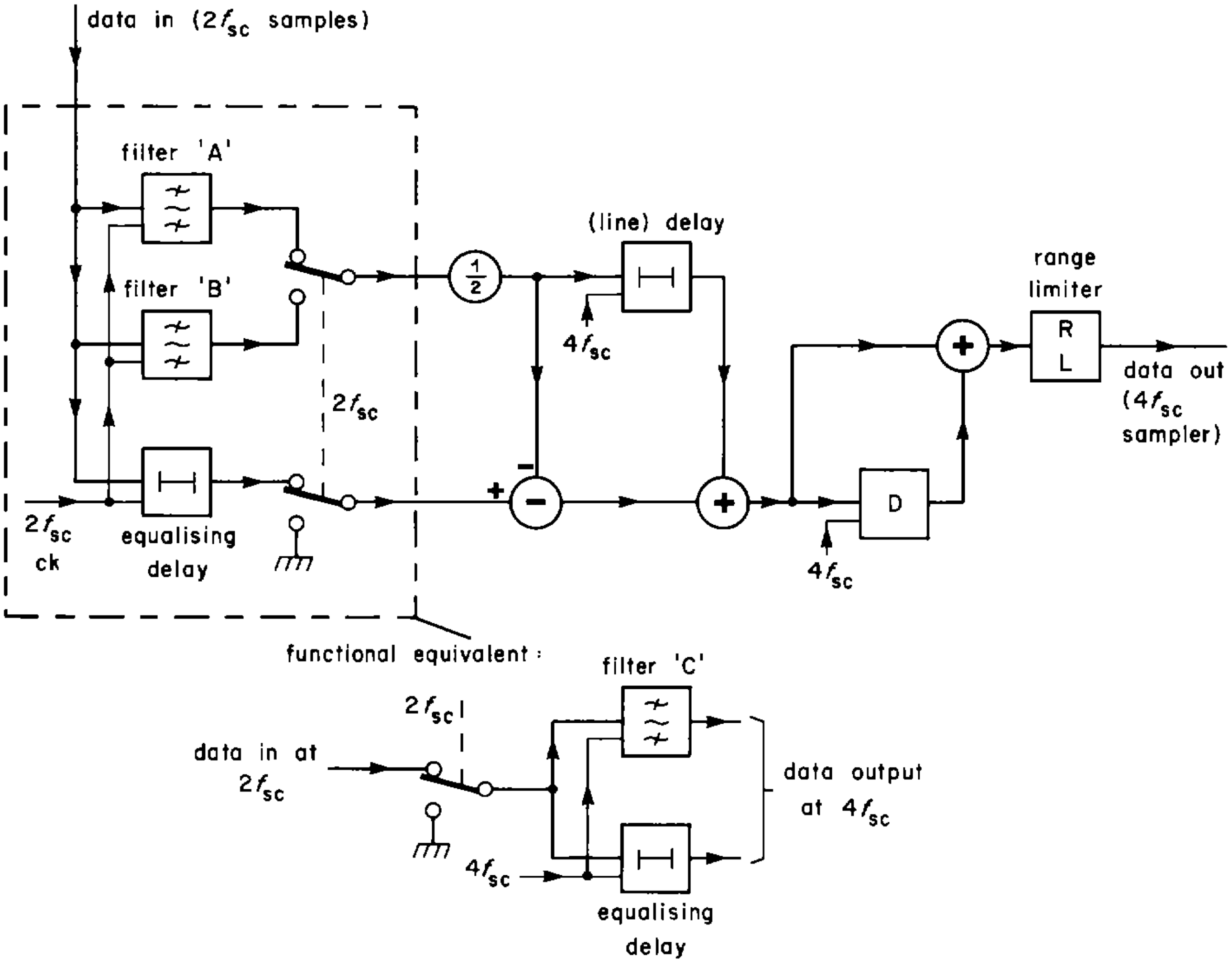

Fig. 16 – Functional block diagram of comb post-filter suitable for cascaded codings and decodings.

screen at a constant speed of approximately 1 picture width/sec. The train and carriages contained areas with fine detail and areas of highly saturated colour.

(ii) Shirt and tie; this test sequence consisted of a green striped shirt and woven tie oscillating slowly ($\pm\frac{1}{4}$ picture widths with a period of approximately 7 sec.) across the screen.

7.4. Test procedure

During the series of subjective tests, test pictures were shown to several different viewing groups each having six observers. The viewers were seated at four and six times picture height from the monitor and the viewing conditions were those of CCIR Recommendation 500–1. The observers were all engineers most of whom were experienced in the critical assessment of television pictures.

At the beginning of each test session the observers were told the type of impairment to be expected and they were asked to grade pictures according to picture clarity and freedom from unwanted effects. A continuous grading scale was used based on the CCIR 5-grade quality scale. The scale appeared on the test forms as

Excellent
Good
Fair
Poor
Bad

and the observers marked this continuous scale according to their assessment.

Two types of test presentation sequence were employed. Initially, as proposed in CCIR Report

405–3, each test condition was presented in the following manner:

10 sec.	RGB reference picture ('Excellent' quality)
3 sec.	Mid grey
15 sec.	Picture corresponding to one of test conditions
10 sec.	Mid grey (voting period)

However, by comparing PAL systems with an RGB reference, subtle differences in PAL picture quality tended to be obscured by the large basic difference in quality between RGB and PAL pictures. Therefore, this type of presentation was changed after the first test to a 'double stimulus' sequence in which:

(i) The test conditions for each slide were presented in pairs as shown:

10 sec.	Condition A
10 sec.	Condition B
10 sec.	Condition A
10 sec.	Condition B

One of the conditions, A or B, in each test was an unprocessed analogue PAL picture but its position within the pair was changed in a random order not known to the viewers.

(ii) The viewers were asked to grade condition A and condition B separately according to the continuous 5-grade quality scale as before. Although the viewers were able to make comparisons between conditions, they were not specifically asked to do so.

As there were many different conditions to be tested, the tests were arranged in six groups. These groups are summarised in Table 1.

Each test condition with each slide was shown twice, in random order, during a test session. A further three or four representative test conditions were added at the start of each session but these were not included in the analysis of the results.

7.5. Results

The mean grades for the different coding processes for each slide are presented in Figs. 17–22. In these figures, the lines joining grade values indicate a common coding process. It should be noted that these figures are not 'graphs' but are a convenient means of comparing the average grades for different slides.

The figures also show two standard errors for the results. These are:

(i) the standard error of the mean grade for each condition ($\sigma/\sqrt{N}$),*

(ii) the standard error of the mean difference (as assessed by each observer) in grades between the processed picture and the analogue reference ($\sigma/\sqrt{N}_{(diff.)}$).

In the figures, only typical values of the standard error are given for clarity. The two standard errors are shown because the spread in results was not primarily due to random variations in a single observer's assessment of the grade of a particular picture but was the result of variations in the average level of grading from different observers. However, observers tended to agree in their assessments of the relative quality of different test conditions. Therefore, $\sigma/\sqrt{N}_{(diff.)}$ is typically less than $\sigma/\sqrt{N}$.

7.6. Discussion of results

7.6.1. Static colour pictures

Figs. 17 and 18 show that for both un-notched and notched PAL signals, the sub-Nyquist systems produce little change (0.1 grade) in the PAL picture quality. Also, it appears that field-delay comb filtering slightly improves the subjective quality (by removing some cross-colour effects).

In Test 2, un-notched analogue PAL was included as one of the test conditions (for comparison with the notched PAL reference). Fig. 18 shows that notched PAL colour pictures were consistently preferred to un-notched PAL colour pictures.

In these static tests, picture-delay comb filtering was not included as one of the sub-Nyquist test conditions as the processed picture was considered indistinguishable from the analogue input signal.

7.6.2. Static monochrome pictures

The results for PAL signals viewed in monochrome are shown in Figs 19 and 20.

Fig. 19 shows that both sub-Nyquist systems with un-notched PAL signals introduced a significant impairment in picture quality. This was for two main reasons:

(i) in the absence of cross-colour, residual alias components from the sub-sampling became more apparent, particularly for luminance frequencies near subcarrier frequency;

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*$\sigma = S \times N/(N - 1)$ where S = standard deviation of test sample and N = number of observers.

Test No.	Static/ Moving	Colour/ Monochrome	Total no. of observers	Reference picture (analogue)	Test conditions
1	Static	Colour	18	RGB	RGB Un-notched PAL Un-notched PAL + line-delay sub-Nyquist Un-notched PAL + field-delay sub-Nyquist
2	Static	Colour	18	Notched PAL	Notched PAL (analogue) Notched PAL + line-delay sub-Nyquist Notched PAL + field-delay sub-Nyquist Un-notched PAL (analogue)
3	Static	Monochrome	12	Un-notched PAL	Un-notched PAL (analogue) Un-notched PAL + line-delay sub-Nyquist Un-notched PAL + field-delay sub-Nyquist
4	Static	Monochrome	12	Notched PAL	As Test 2
5	Moving	Colour and Monochrome	12	Un-notched PAL	Un-notched PAL (analogue) Un-notched PAL + field-delay sub-Nyquist Un-notched PAL + picture-delay sub-Nyquist
6	Moving	Colour and Monochrome	12	Notched PAL	As 5 but with notched PAL coder

Table 1 : Summary of subjective test grouping

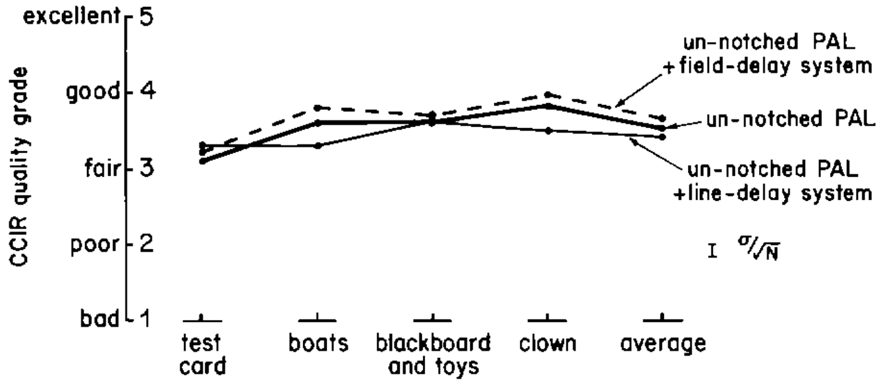


Fig. 17 – Test 1 results for colour pictures (un-notched PAL + sub-Nyquist systems).

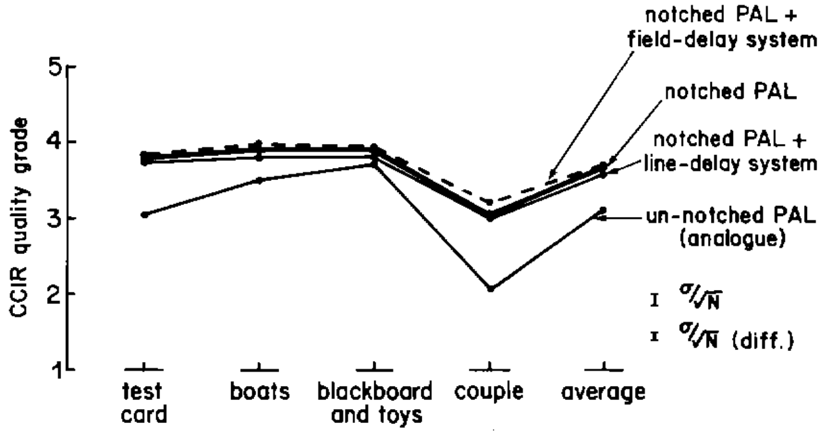


Fig. 18 – Test 2 results for colour pictures (notched PAL + sub-Nyquist systems).

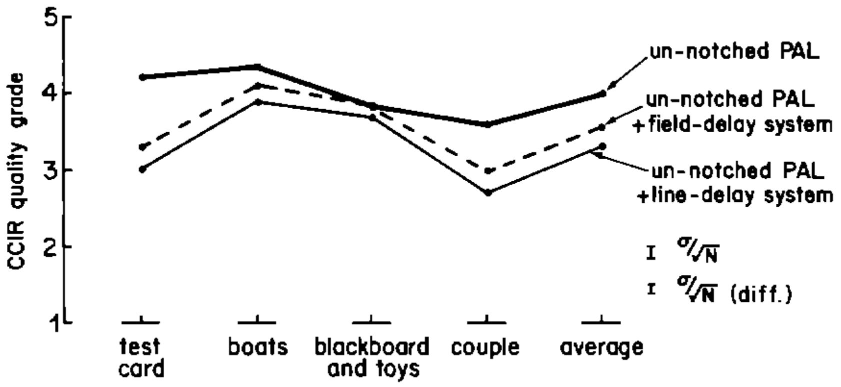


Fig. 19 – Test 3 results for monochrome pictures (un-notched PAL + sub-Nyquist systems).

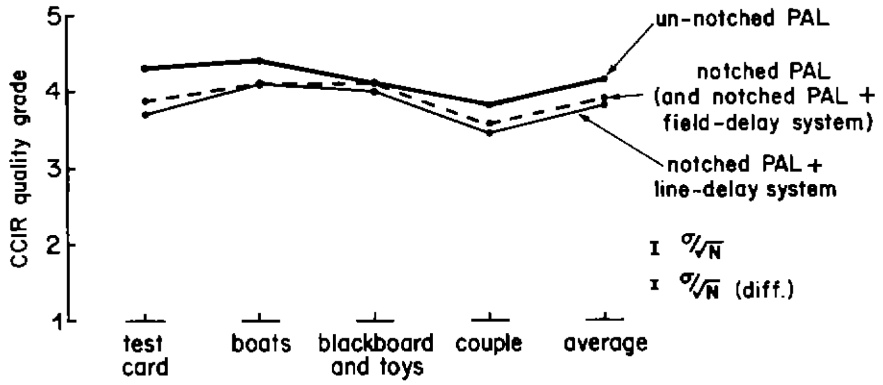


Fig. 20 – Test 4 results for monochrome pictures (notched PAL + sub-Nyquist systems).

(ii) without cross-colour effects on diagonal luminance transitions, the slight loss of diagonal resolution was more visible.

However, Fig. 20 shows that for notched PAL signals a line-delay sub-Nyquist system introduces little (0.1 grade) impairment and a field-delay system introduces no measurable impairment in picture quality. This test also showed that, in monochrome, un-notched signals were preferred to notched signals.

7.6.3. Moving pictures

Figs. 21 and 22 show the subjective gradings for the two movement sequences for field-delay and picture-delay based sub-Nyquist systems. For this test material, preliminary tests showed that line-delay based sub-Nyquist systems did not introduce any noticeable impairment to the PAL picture quality.

Both picture- and field-delay comb filters introduced significant picture impairment with movement, in particular picture-delay comb filters with the model train sequence. In this sequence with areas of highly-saturated colour, the averaging of chrominance information between pictures caused a very obvious smearing of colour across chrominance boundaries.

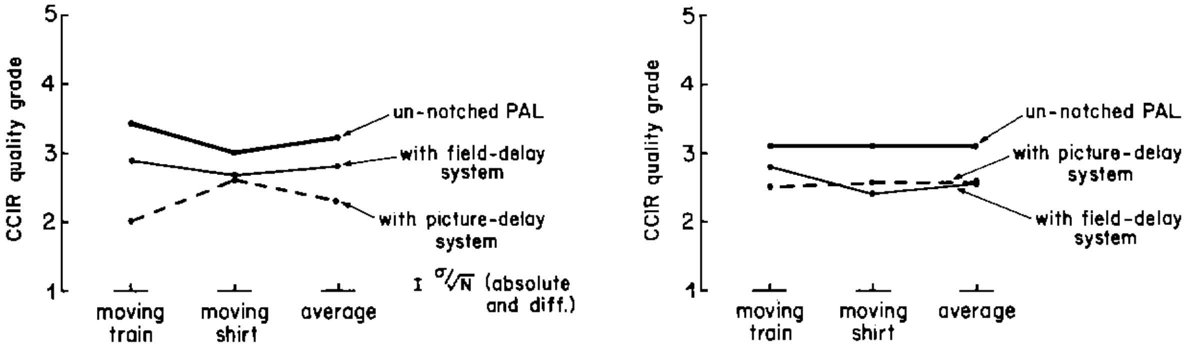
On average, picture-delay comb filters introduced a greater picture impairment than field-delay comb filters. However, for particular spatial frequencies and speeds of movement, field-delay combing could give the greater picture impairment. This was the case for the moving shirt sequence when viewed in monochrome.

7.7. Summary of results

Various sub-Nyquist systems were tested in the following way:

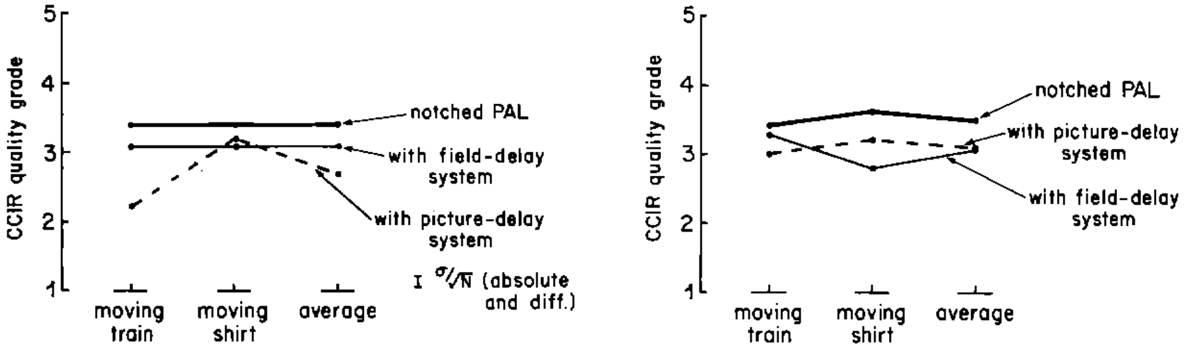
(i) Static picture tests: sub-Nyquist sampling with line- and field-delay based comb filters;
(ii) Moving picture tests: sub-Nyquist sampling with field- and picture-delay based comb filters.

The tests were performed for both notched and un-notched PAL signals. The results are summarised in Tables 2 and 3.



(a) (b)

Fig. 21 – Test 5 results; sub-Nyquist system grades for moving pictures with un-notched PAL signals: (a) Colour pictures; (b) Monochrome pictures.



(a) (b)

Fig. 22 – Test 6 results; sub-Nyquist system grades for moving pictures and notched PAL signals: (a) Colour pictures; (b) Monochrome pictures.

	Static Pictures				Moving Pictures			
	Analogue	Sub-Nyquist system			Analogue	Sub-Nyquist system		
		Line	Field	Picture*		Line*	Field	Picture
Colour	3.65	3.55	3.7	3.65	3.4	3.4	3.1	2.7
Monochrome	3.9	3.8	3.9	3.9	3.5	3.5	3.1	3.1

*Preliminary tests only.

Table 2 : Average grades for notched PAL with sub-Nyquist systems

	Static Pictures				Moving Pictures			
	Analogue	Sub-Nyquist system			Analogue	Sub-Nyquist system		
		Line	Field	Picture		Line*	Field	Picture
Colour	3.5	3.4	3.65	3.5	3.2	3.2	2.8	2.3
Monochrome	4.0	3.3	3.55	4.0	3.1	3.1	2.6	2.6

*Preliminary tests only.

Table 3 : Average Grades for un-notched PAL with sub-Nyquist systems

8. Conclusions

In a sub-Nyquist system for PAL signals, different comb-filter configurations are possible based on line, field or picture delays. Experimental digital comb filters were constructed and their design and theory of operation are described. A series of subjective tests was carried out to compare and quantify the impairments introduced by the different systems. On balance, it was found that line-delay comb filters gave the best overall picture quality. Although field- and picture-delay comb filters did not introduce any subjective impairment to static colour pictures they gave a noticeable impairment with movement, making them less suitable, overall, for broadcast use.

A sub-Nyquist system using digital, line-delay comb filters introduced on average only 0.1 grade impairment to PAL colour pictures which were themselves graded with an average quality rating of about 3.5. These figures were obtained using a continuous quality grading assessment based on the 5-grade CCIR quality scale. A line-delay-based system produced a somewhat greater impairment when the signal had been coded using an 'un-notched' PAL coder and viewed in monochrome without a notch at subcarrier frequency in the monitor luminance channel. This is not, however, considered to be a serious disadvantage.

The design of the experimental filters would not be ideal in systems where multiple sub-Nyquist systems might be cascaded. Appropriate design modifications, allowing for cascading, have been described. For a single coding and decoding, these modifications would not give a perceptible improvement in picture quality. Finally, a figure for the quantisation noise introduced by a digital sub-Nyquist transmission system has been derived. For a typical system the unweighted and weighted signal-to-noise ratios introduced by the digital coding are approximately 51.0 dB and 57.8 dB respectively.

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Appendix A: Derivation of chrominance vertical frequency responses for sub-Nyquist systems

A.1. Notation

When a signal with a frequency spectrum $S(f)$ is modulated onto a subcarrier by multiplying by $2\cos(2\pi f_{sc}+\theta)$, the modulated signal may be represented by

$$S(f)_{\text{modulated}} = S^{-1}\mathrm{e}^{-\mathrm{j}\theta} + S^{+1}\mathrm{e}^{+\mathrm{j}\theta}$$

where $S^{+1} = S(f-f_{sc})$ and is the original baseband signal shifted *up* in frequency by f_{sc}.

Similarly when a signal $S(f)$ is sampled at a frequency $2f_{sc}$, at instants $t=\tau, \tau+T_{sc}/2, \tau+T_{sc}, \ldots$, where $T_{sc}=f_{sc}^{-1}$, the spectrum of the sampled signal is given by

$$S(f)_{\text{sampled}} = \cdots + S^{-4}\mathrm{e}^{-\mathrm{j}4\phi} + S^{-2}\mathrm{e}^{-\mathrm{j}2\phi} + S + S^{+2}\mathrm{e}^{\mathrm{j}2\phi} + S^{+4}\mathrm{e}^{\mathrm{j}4\phi} + \cdots$$

where $$S^{+n} = S(f-nf_{sc})$$

and $$\phi = \frac{-2\pi\tau}{T_{sc}} \qquad \text{(A.1)}$$

$S(f-nf_{sc})$ is the spectrum $S(f)$ shifted up in frequency by nf_{sc}.

A.2. Chrominance path

Consider the overall path from PAL coder to PAL decoder as shown in Fig. A1. If $S(f)$ is the input chrominance signal then the signals at the various stages will be given by the following:

(i) At output of PAL modulator:

$\frac{1}{2}(S^{+1}\mathrm{e}^{-\mathrm{j}\zeta} + S^{-1}\mathrm{e}^{-\mathrm{j}\zeta})$,

where ζ = phase of subcarrier at $t=0$.

(ii) After pre-filtering: $\frac{1}{2}F(S^{+1}\mathrm{e}^{\mathrm{j}\zeta} + S^{-1}\mathrm{e}^{-\mathrm{j}\zeta})$.

(iii) After sub-sampling:

$$\frac{1}{2}F(S^{+1}\mathrm{e}^{\mathrm{j}\zeta} + S^{-1}\mathrm{e}^{-\mathrm{j}\zeta}) + \frac{1}{2}F^{+2}(S^{+3}\mathrm{e}^{\mathrm{j}(\zeta+2\phi)} + S^{+1}\mathrm{e}^{-\mathrm{j}(\zeta-2\phi)}) + \cdots + \frac{1}{2}F^{-2}(S^{-1}\mathrm{e}^{\mathrm{j}(\zeta-2\phi)} + S^{-3}\mathrm{e}^{-\mathrm{j}(\zeta+2\phi)}) + \cdots$$

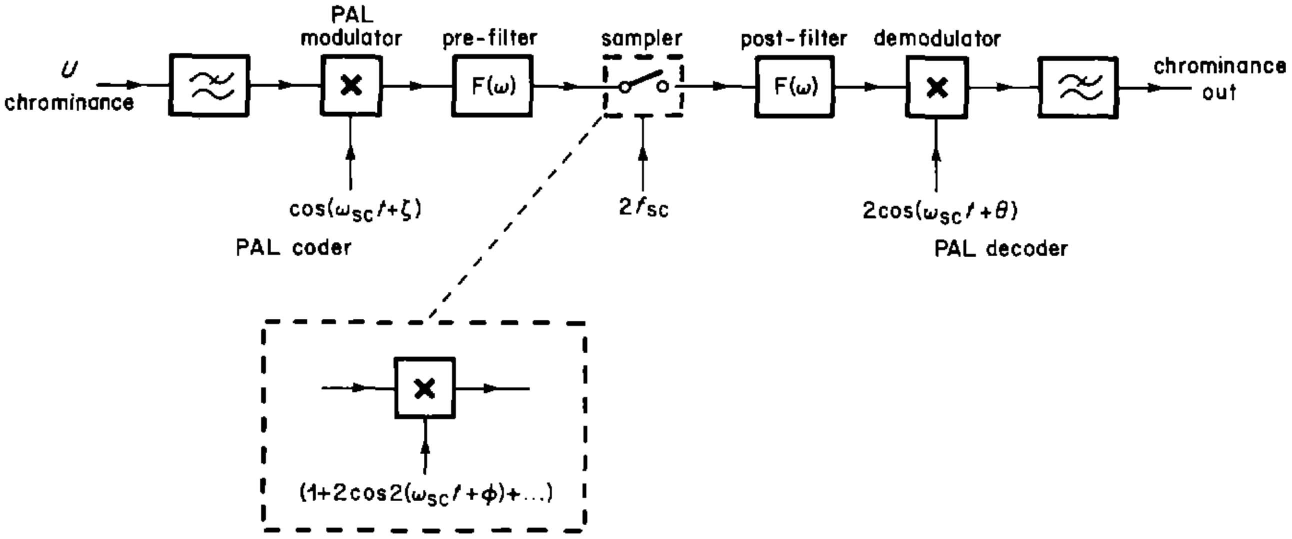

Fig. A.1 – U chrominance path.
PAL coder-sub-Nyquist system-PAL decoder.

(iv) After post-filtering:

$$\tfrac{1}{2}FF(S^{+1}\mathrm{e}^{\mathrm{j}\zeta} + S^{-1}\mathrm{e}^{-\mathrm{j}\zeta}) + \tfrac{1}{2}FF^{+2}S^{+1}\mathrm{e}^{-\mathrm{j}(\zeta-2\phi)} + \tfrac{1}{2}FF^{-2}S^{-1}\mathrm{e}^{\mathrm{j}(\zeta-2\phi)}$$

since terms such as $FS^{+3} = FS^{-3} = 0$ for band-limited signals.

(v) After demodulation and low-pass filtering the spectrum is

$$\tfrac{1}{2}(F^{+1}F^{+1}\mathrm{e}^{-\mathrm{j}(\zeta-\theta)} + F^{-1}F^{-1}\mathrm{e}^{\mathrm{j}(\zeta-\theta)})S(f) + \tfrac{1}{2}F^{-1}F^{+1}(\mathrm{e}^{-\mathrm{j}(\zeta-2\phi+\theta)} + \mathrm{e}^{\mathrm{j}(\zeta-2\phi+\theta)})S(f) \qquad \text{(A.2)}$$

since terms containing S^{+2} and S^{-2} etc. are removed by the low-pass filtering.

Over the chrominance band the pre- and post-filter responses are given by $F(\omega) = \frac{1}{2}(1 + \mathrm{e}^{-\mathrm{j}\omega T})$ where T is the length of the comb filter delay element. The chrominance response of the system may then be determined from Equation (A.2) by assighing the appropriate values of T, ζ, ϕ and θ.

A.3. Line-delay-based comb filtering

For a line-delay comb filter, $\omega_{\mathrm{sc}}T = -\pi/2$, then

$$F^{+1} = F(\omega - \omega_{\mathrm{sc}}) = \tfrac{1}{2}(1 - \mathrm{j}\mathrm{e}^{-\mathrm{j}\omega T})$$

and $$F^{-1} = F(\omega + \omega_{\mathrm{sc}}) = \tfrac{1}{2}(1 + \mathrm{j}\mathrm{e}^{-\mathrm{j}\omega T})$$

For the U component

$$\zeta = 0$$

$$\phi = \frac{-2\pi T}{2T_{\mathrm{sc}}} = (\text{integer} \times 2\pi) + \frac{\pi}{4} \equiv \frac{\pi}{4}$$

and $\theta = -\omega_{\mathrm{sc}}T \equiv \dfrac{\pi}{2}$

This value of ϕ means that the sampling is in phase with the U sub-carrier after the group-delay of $T/2$ of the pre-filter, and the value of θ ensures synchronous demodulation of the U component after the group delay, T, of the pre- and post-filter together. Then Equation (4.2) gives, for the U output, $S_{uu}(\omega)$:

$$\begin{aligned} S_{uu}(\omega) &= \tfrac{1}{2}\{\tfrac{1}{4}\mathrm{j}(1 - \mathrm{j}\mathrm{e}^{-\mathrm{j}\omega T})^2 - \tfrac{1}{4}\mathrm{j}(1 + \mathrm{j}\mathrm{e}^{-\mathrm{j}\omega T})^2\}S_u(\omega) \\ &\quad + \tfrac{1}{2}\{\tfrac{1}{4}(1 + \mathrm{j}\mathrm{e}^{-\mathrm{j}\omega T})(1 - \mathrm{j}\mathrm{e}^{-\mathrm{j}\omega T})\}2S_u(\omega) \\ &= \tfrac{1}{2}\mathrm{e}^{-\mathrm{j}\omega T}(1 + \cos\omega T)S_u(\omega) \qquad \text{(A.3)} \end{aligned}$$

where $S_u(\omega)$ is the spectrum of the U component at the input to the PAL coder and $S_{uu}(\omega)$ refers to that proportion of the output U spectrum which is derived from the input U component.

Thus, the U component is delayed by a time T and has an amplitude $\frac{1}{2}(1 + \cos\omega T)$. Since for stationary pictures

$$f = (\mathrm{m} - 2\mathrm{n}/\mathrm{N})f_{\mathrm{L}} \qquad \text{where} \qquad f_{\mathrm{L}} = 1/T,$$

then $$S_{uu}(\omega) = \tfrac{1}{2}\mathrm{e}^{-\mathrm{j}\omega T}\left(1 + \cos 2\pi\frac{2\mathrm{n}}{625}\right)S_u$$

It is only the vertical frequency component, n, that gives rise to attenuation. This vertical frequency response is the same as that shown in Fig. 5.

For the V component, ζ, 2ϕ and θ have the following values

$$\zeta = \frac{\pi}{2}$$

$$2\phi = \frac{\pi}{2} \text{ as before}$$

$$\theta = \frac{\pi}{2} + \zeta = \pi$$

Then Equation (A.2) gives, for the V component, after demodulation:

$$S_{vv}(\omega) = \tfrac{1}{2}\mathrm{e}^{-\mathrm{j}\omega T}(1 - \cos\omega T)S_v(\omega)$$

This expression shows that the frequency response for the V signal has a maximum at 156c./p.h., where the V information is centred (before inversion of the V-axis switch).

Equation (A.2) can also be used to give the magnitude of the U/V cross effects. For example, in calculating the U to V cross-talk ζ, 2ϕ and θ take the values $\zeta = 0$, $2\phi = \pi/2$ and $\theta = 0$ (or π). Equation (A.2) then gives, for the cross term, $S_{uv}(\omega)$, after some reduction

$$S_{uv}(\omega) = \tfrac{1}{2}\mathrm{e}^{-\mathrm{j}\omega T}\sin(\omega T)S_u(\omega) \qquad \text{(A.4)}$$

where ω, here, refers to the U signal component frequency.

In terms of spatial frequency, this gives

$$S_{uv}(\omega) = \tfrac{1}{2}\mathrm{e}^{-\mathrm{j}\omega T}\sin\left(\frac{4\pi\mathrm{n}}{625}\right)S_u(\omega)$$

The magnitude of the cross-talk therefore depends on the vertical frequency component in the manner shown in Fig. 9.

A.4. 'Field'-delay-based filtering

In the field-delay case where $T = 313$ lines, $\omega_{sc}T + \pi/2$ and, for the U component, $\zeta = 0$, $2\phi = -\pi/2$ and $\theta = -\pi/2$. Also $F^{+1} = \frac{1}{2}(1 + \mathrm{j}e^{-\mathrm{j}\omega T})$ and $F^{-1} = \frac{1}{2}(1 - \mathrm{j}e^{-\mathrm{j}\omega T})$.

Substituting these values into Equation (A.2) we again obtain Equations (A.3) for the U chrominance response. Changing the value of θ to 0 (or π) again gives Equation (A.4) for the U to V cross talk.

For stationary pictures

$$\omega T_{\text{field}} = 2\pi\left(\mathrm{m} - \frac{2\mathrm{n}}{625}\right)313$$

$$= (\text{integer} \times 2\pi) + \frac{2\pi \mathrm{n}}{625}$$

Then Equations (A.3) and (A.4) give

$$|S_{uu}(\mathrm{m},\mathrm{n})| = \left(1 + \cos\frac{2\pi\mathrm{n}}{625}\right)|S_u(\mathrm{m},\mathrm{n})|$$

and

$$|S_{uv}(\mathrm{m},\mathrm{n})| = \left|\sin\left(\frac{2\pi\mathrm{n}}{625}\right)\right| |S_u(\mathrm{m},\mathrm{n})|$$

This variation in vertical chrominance resolution and the magnitude of the U/V cross term for stationary pictures are also plotted in Fig. 9.

Following the same procedure for the case of picture-delay-based filtering shows that, since for stationary pictures $\omega T_{\text{picture}}$ equals an integer multiple of 2π, there is no U/V cross talk or loss of vertical chrominance resolution for stationary pictures. This does not hold with picture movement.

Appendix B. Comb-filtering at the transition between combed and uncombed regions, for cascaded sub-Nyquist systems

For filters of the form shown in Fig. 3, the response $G(\omega)$ of the pre-filter and post-filter to a signal $e^{\mathrm{j}\omega T}$ are respectively:

$$G_{\text{pre}}(\omega) = L_2(\omega)\left[L_1(\omega)e^{-\mathrm{j}\omega T} + (1 - L_1(\omega))e^{-\mathrm{j}\omega T/2}\cos\frac{\omega T}{2}\right]$$

and

$$G_{\text{post}}(\omega) = L_2(\omega)\left[L_1(\omega) + (1 - L_1(\omega))e^{-\mathrm{j}\omega T/2}\cos\frac{\omega T}{2}\right]$$

(L_1 and L_2 are assumed to have uniform group delay which can be ignored here).

For cascaded sub-Nyquist coding, there is a combination of a post-filter followed by a pre-filter between successive samplings. Then, the overall frequency response $G(\omega)$ between samplings is given by

$$\begin{aligned} G(\omega) &= G_{\text{post}}(\omega)G_{\text{pre}}(\omega) \\ &= L_2{}^2\left[L_1{}^2 + (1 - L_1)^2\cos^2\frac{\omega T}{2} + 2L_1(1 - L_1)\cos^2\frac{\omega T}{2}\right]e^{-\mathrm{j}\omega T} \\ &= L_2{}^2\left[L_1{}^2 + (1 - L_1{}^2)\cos^2\frac{\omega T}{2}\right]e^{-\mathrm{j}\omega T} \end{aligned}$$

This response has a uniform group delay, T, at all frequencies, and may be considered to fall into five regions. These are

(i) low frequency region: $L_2 = 1$ and $L_1 = 1$
(ii) lower transition band: $L_2 = 1$ and $1 > L_1 > 0$
(iii) fully combed region: $L_2 = 1$ and $L_1 = 0$
(iv) upper transition band: $1 > L_2 > 0$ and $L_1 = 0$
(v) out-of-band region: $L_2 = 0$ and $L_1 = 0$

On sampling, a frequency ω lying in region (ii) will give rise to an alias at $2\omega_{sc} - \omega$ lying in region (iv). For cascadability we require to find the condition on L_2 and L_1 such that

$$|G(\omega)| + |G(2\omega_{sc} - \omega)| = 1$$

For ω in region (ii)

$$\begin{aligned} &|G(\omega)| + |G(2\omega_{sc} - \omega)| \\ &\quad = L_1{}^2(\omega) + (1 - L_1{}^2(\omega))\cos^2\frac{\omega T}{2} + L_2{}^2(2\omega_{sc} - \omega)\cos^2\frac{(2\omega_{sc} - \omega)T}{2} \\ &\quad = L_1{}^2(\omega) + (1 - L_1{}^2(\omega))\cos^2\frac{\omega T}{2} + L_2{}^2(2\omega_{sc} - \omega)\sin^2\frac{\omega T}{2} \end{aligned}$$

since $\omega_{sc}T = \pm\pi/2$

Therefore, $|G(\omega)| + |G(2\omega_{sc} - \omega)| = 1$

if $1 - L_1{}^2(\omega) = L_2{}^2(2\omega_{sc} - \omega)$

which is the condition stated in Equation 5. Similarly, the same condition holds for a frequency ω falling initially in region (iv).

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